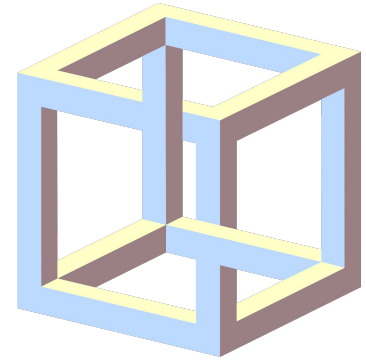


Meschers: Geometry Processing of Impossible Objects



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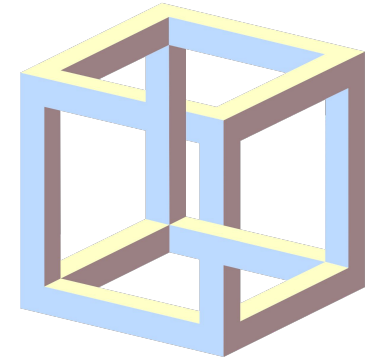
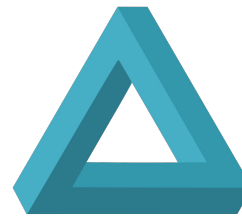
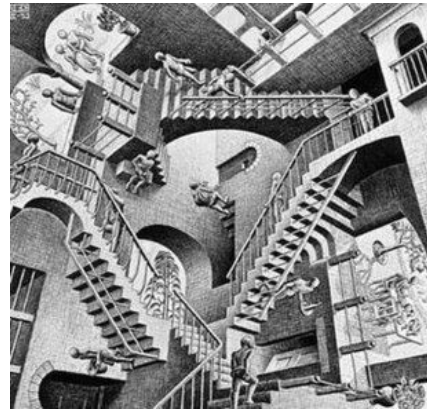
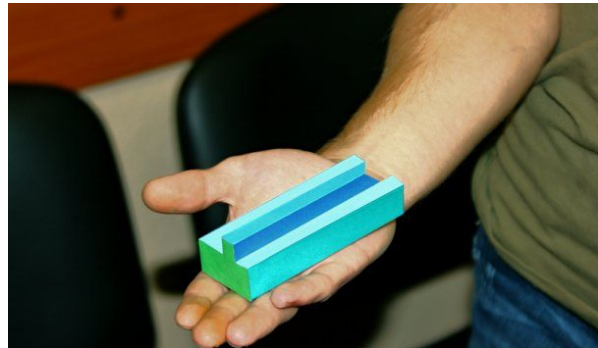


Contexte - Objects Impossibles

Oscar Reutersvärd (1915-2002)

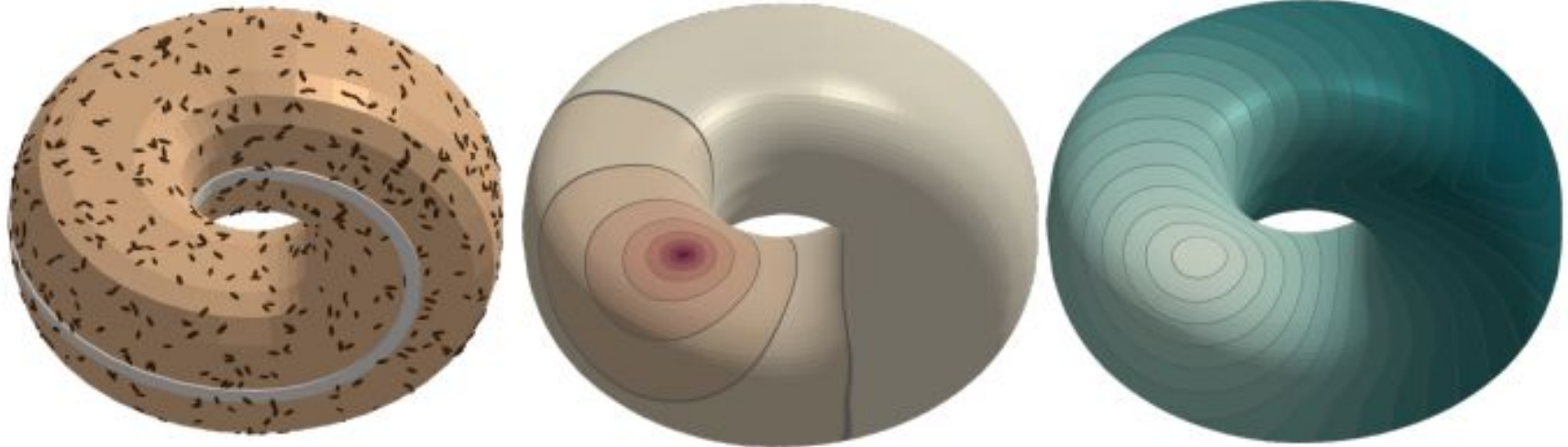
Maurits Cornelis Escher's woodcuts

Paradoxe - Contraire aux lois de la physique



Objectifs (en quelques mots)

Rendre et pouvoir calculer sur ce genre d'objets !



Problématique

Intégration de l'information locale dans une géométrie globale => Problèmes

“What makes the object impossible is that one cannot assign an absolute depth to every point on its surface without cutting or bending the geometry, which would violate the local depth cues.”



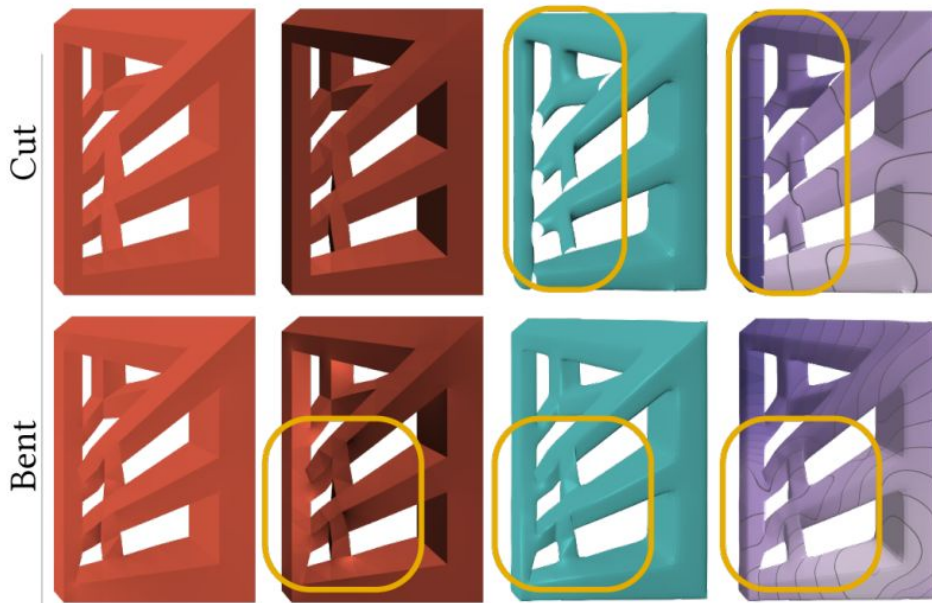
Prior work :

=> Cutting

=> Bending

Often breaks the geometry processing pipeline

La solution : MESCHERS



Meshers ?

Même topologie qu'un maillage triangulaire orienté avec ou sans "boundary".

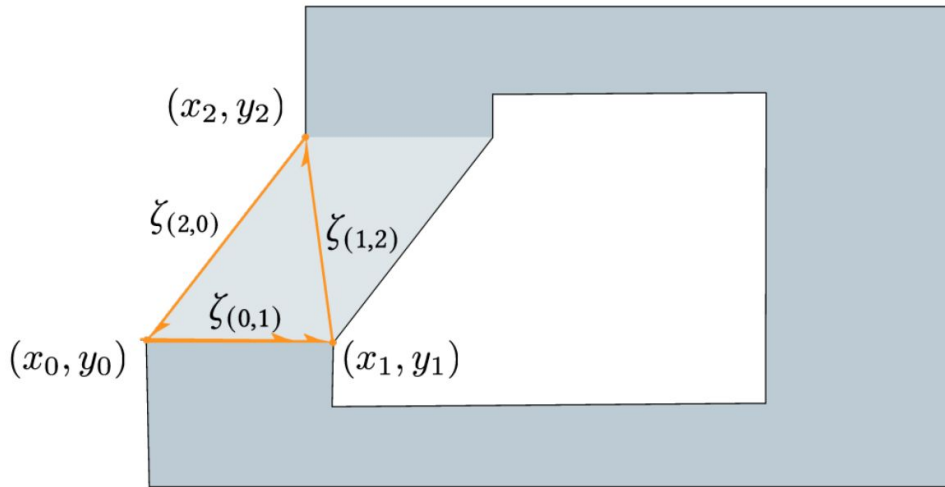
Un set de faces triangulaires **F**, arêtes **E** et sommets **V**.

Une orientation pour chaque faces et arêtes.

Ce qui change :

=> Coordonnées screen-space $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{|\mathbf{V}|}$

=> Profondeur relative stockées sur les arêtes $\boldsymbol{\zeta} \in \mathbb{R}^{|\mathbf{E}|}$
(Distance signée entre p et q)

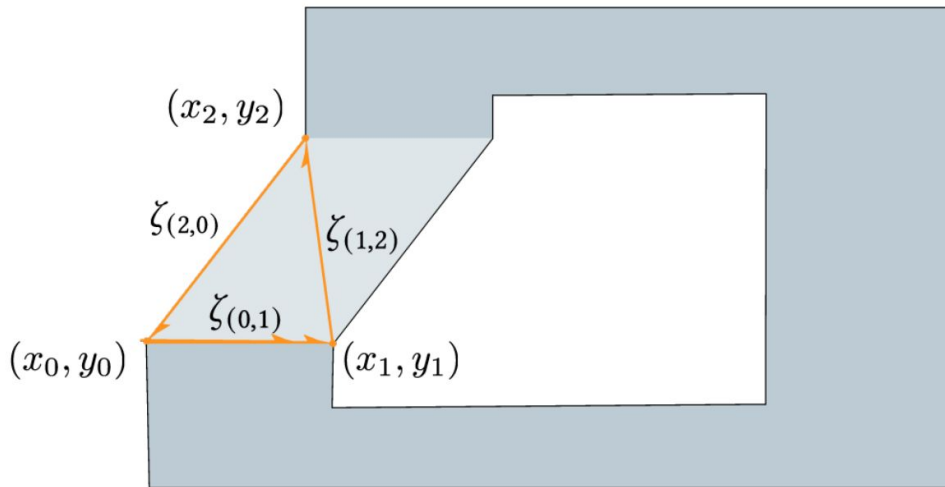


Meshers ?

Discrete Exterior Calculus (DEC)

$\mathbf{x}$ et $\mathbf{y}$ primal 0-form (Un scalaire par sommet)

ζ primal 1-form (Une valeur par arête)



Discrete Exterior Derivatives (Primal)

(Entre 3 arêtes
d'une face)

$$d_{12} \in \{-1, 0, 1\}^{|F| \times |E|}$$

(Entre 2 sommets
d'une arête)

$$d_{01} \in \{-1, 0, 1\}^{|E| \times |V|}$$

Exterior Derivatives (Dual)

$$d_{12}^T \text{ and } d_{01}^T$$

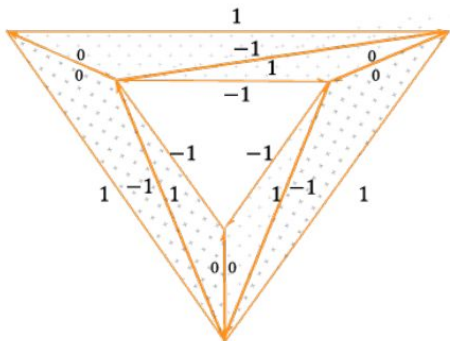
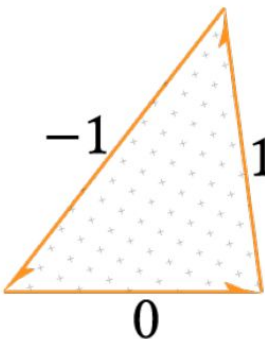
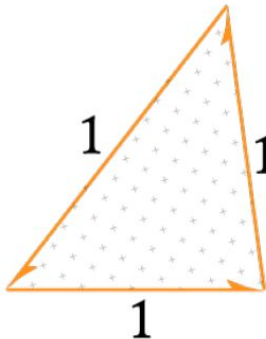
Intégrabilité

$d_{12}\zeta$ Somme orientée des 1-forms autour du triangle

$d_{12}\zeta = 0$ Local integrability != Global Integrability

Local integrability != Global Integrability

Discrete de Rham cohomology
 ★0 (one-ring barycentric areas),
 ★1(cotangent weights), and
 ★2(inverse triangle areas)



Depth Ordering

Structure supplémentaire :
Graphe acyclique

Choix de l'utilisateur (selection groupe)

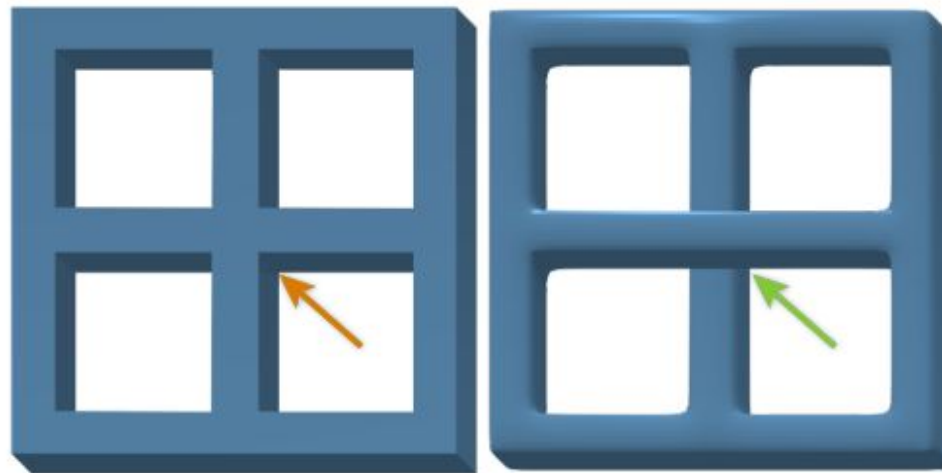


Fig. 4. WINDOW. This mescher is globally integrable: naïvely rendering it makes the bars intersect at the same depth (left). To create an impossibility, an additional form of depth ordering is necessary (Section 3.4). After introducing depth ordering, we can render WINDOW as an impossible object. Here, we further subdivide the mescher and apply our smoothing operator to create specular highlights that emphasize the impossible depth offset.

Rendering

Flattening

Use per-face and per-vertex normals derived from depth representation

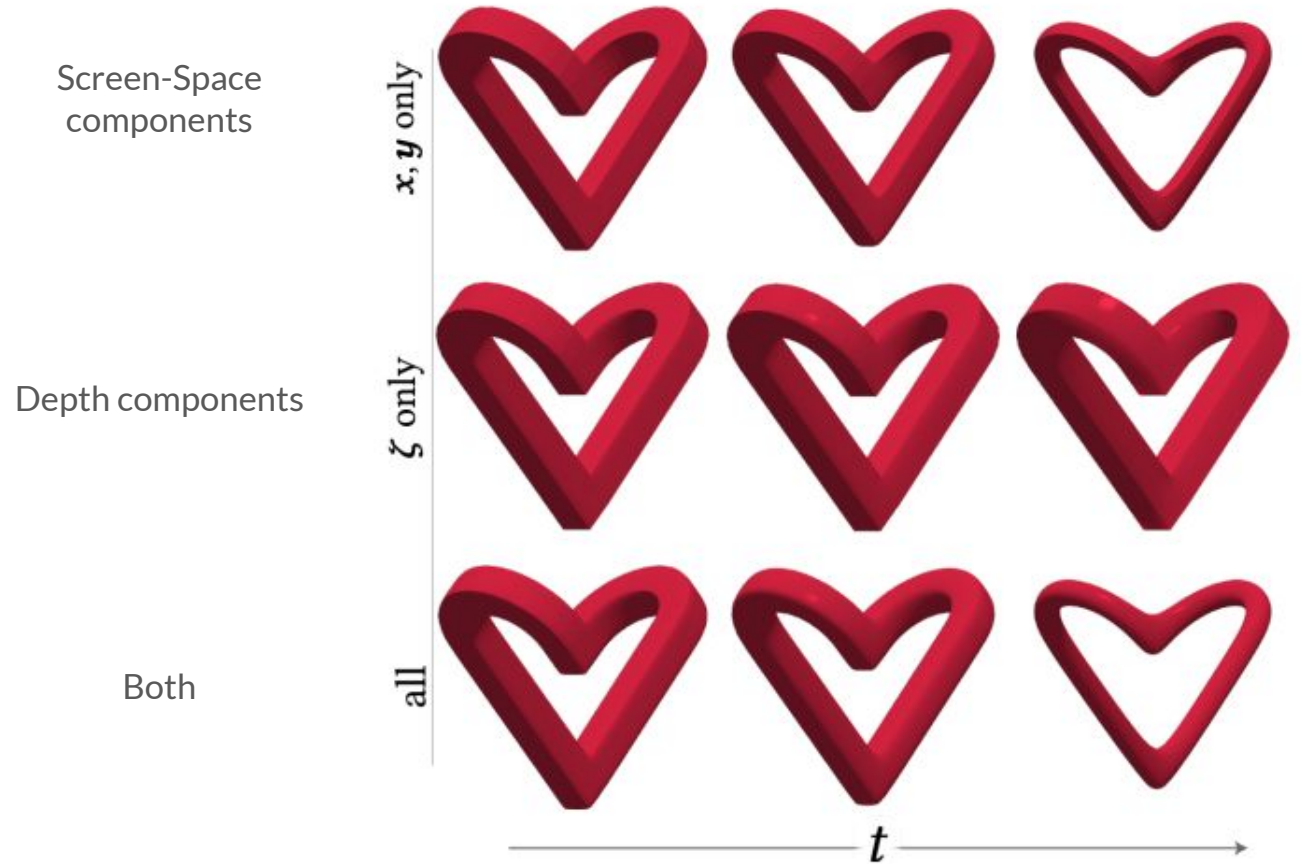
Contraintes :

Only directional or
Environment map illumination

(Area lights require 3D position...)



Smoothing

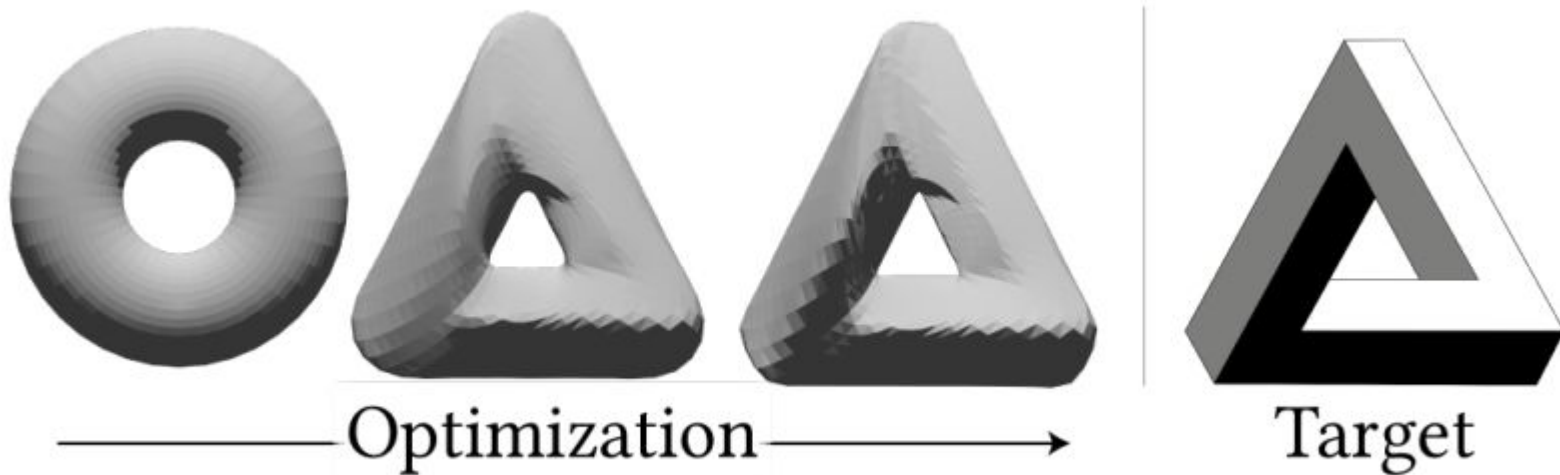


Inverse Rendering

Sobolev gradient descent (Smooth sparse gradients over the surface)

SoftRas differentiable rasterizer $\hat{g} = (I + \lambda \Delta)^{-1} g$

(La structure de depth ordering est remplacée par une structure globale différentiable)



Heat diffusion / geodesic distances

Discrete de Rham cohomology
★0 (one-ring barycentric areas),
★1(cotangent weights), and
★2(inverse triangle areas)

Elements finis :

Gradients par face : $G_x, G_y, G_z \in \mathbb{R}^{|\mathbf{F}| \times |\mathbf{V}|}$

Diagonal area matrix : $A \in \mathbb{R}^{|\mathbf{F}| \times |\mathbf{F}|}$

$$A = \star_2^{-1} \quad G^T A G = d_{01}^T \star_1$$

Operateur Laplacien Discret

$$\mathbf{u}_0 \in \mathbb{R}^{|\mathbf{V}|} \quad (I + t\Delta)\mathbf{u} = \mathbf{u}_0$$

$$\mathbf{g} = \frac{G\mathbf{u}}{\|G\mathbf{u}\|_2}.$$

$$G^T A G \mathbf{d} = -G^T A \mathbf{g}$$

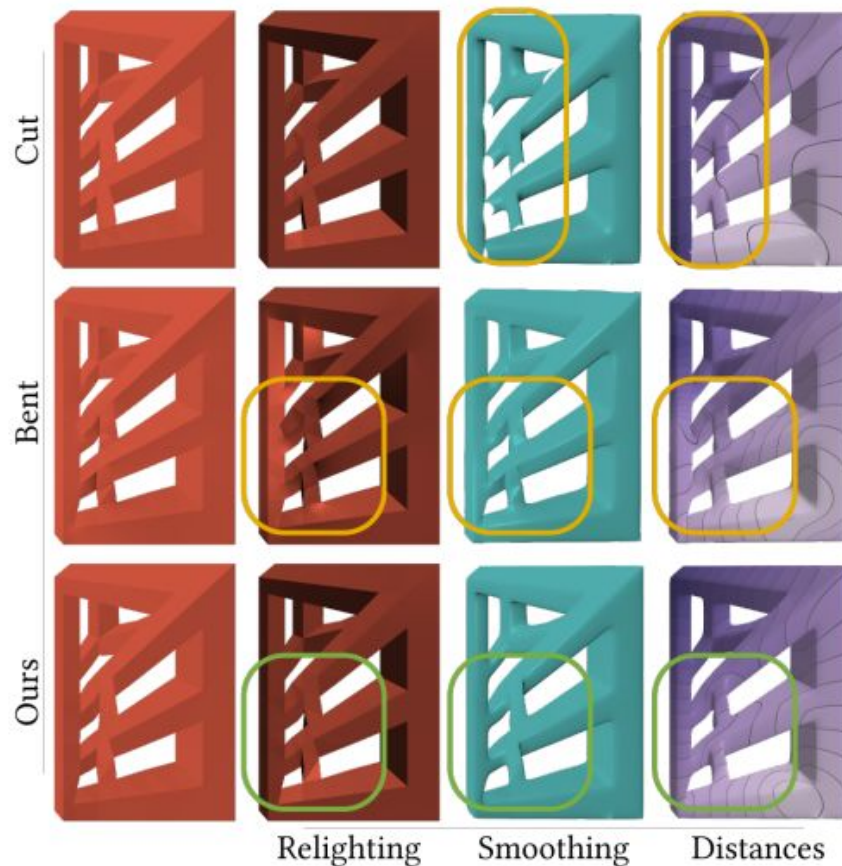


Conclusion

- A geometric representation of impossible objects built on DEC (Discrete Exterior Calculus).
- The rendering and relighting of impossible objects.
- Common modeling operations of smoothing and subdivision.
- Answering distance queries on impossible objects.
- Inverse rendering of impossible objects.

Ma conclusion :

“Représenter des objets impossibles en 3D est impossible.”



Merci !